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The Approximate Solutions of Helmholtz and Coupled Helmholtz Equations on Cantor Sets within Local Fractional Operator

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Abstract

In this paper, we proposed local fractional Laplace decomposition method to solve the Helmholtz and coupled Helmholtz equations on Cantor sets within local fractional operator. The approximate solutions are obtained by using the local fractional Laplace decomposition method, which is the coupling method of local fractional Laplace transform and Adomian decomposition method. Illustrative examples are included to demonstrate the high accuracy and fast convergence of this new method.

Introduction

The Helmholtz equation, named for Hermann von Helmholtz, often arises in the study of physical problems involving partial differential equations such as electromagnetic radiation, seismology, transmission, and acoustics. Kreb and Roach [1] discussed the transmission problems for the Helmholtz equation. Kleinman and Roach [2] studied the boundary integral equations for the three-dimensional Helmholtz equation. Karageorghis [3] presented the eigenvalues of the Helmholtz equation. Fu and Mura [4] suggested the volume integrals of the Helmholtz equation. Samuel and Thomas [5] proposed the fractional Helmholtz equation.

The local fractional Helmholtz equation in two-dimensional case was suggested in [6] as follows:

$$\frac{\partial^{2\alpha} H(x,y)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} H(x,y)}{\partial y^{2\alpha}} + \omega^{2\alpha} H(x,y) = f(x,y),\ 0<\alpha\le 1, \tag{1}$$

with the initial value conditions as follows:

$$H(0,y)=\varphi(y),\quad \frac{\partial^{\alpha} H(0,y)}{\partial x^{\alpha}} = \psi(y), \tag{2}$$

where $H(x,y)$ is unknown function and $f(x,y)$ is a source term.

Recently, the local fractional Helmholtz equation was solved by local fractional variational iteration method [7,8], local fractional series expansion method [8]. In this paper, our aim is to present the coupling method of local fractional Laplace transform and Adomian decomposition method, which is called as the local fractional Laplace decomposition method, and to used it to solve the differential Helmholtz and coupled Helmholtz equations on Cantor sets within local fractional operator.

Local Fractional Laplace Decomposition Method (LFLDM)

Let us consider the following partial differential equation within local fractional derivative [10]:

$$L_\alpha u(x,y) + R_\alpha u(x,y) = f(x,y), \tag{3}$$

where $L_\alpha = \frac{\partial^{k\alpha}}{\partial x^{k\alpha}}$ denotes the linear local fractional differential operator, R_α is the remaining linear operator, and $f(x,t)$ is a source term.

Taking local fractional Laplace transform on Eq. (3), we obtain

$$Ł_\alpha\{L_\alpha u(x,y)\} + Ł_\alpha\{R_\alpha u(x,y)\} = Ł_\alpha\{f(x,y)\}. \tag{4}$$

By applying the local fractional Laplace transform differentiation property, we have

$$s^{k\alpha} Ł_\alpha\{u(x,y)\} - s^{(k-1)\alpha} u(0,y) - s^{(k-2)\alpha} u^{(\alpha)}(0,y) - \cdots - u^{((k-1)\alpha)}(0,y) + Ł_\alpha\{R_\alpha u(x,y)\} = Ł_\alpha\{f(x,y)\}, \tag{5}$$

or

$$Ł_\alpha\{u(x,y)\} = \frac{1}{s^\alpha} u(0,y) + \frac{1}{s^{2\alpha}} u^{(\alpha)}(0,y) + \cdots + \frac{1}{s^{k\alpha}} u^{((k-1)\alpha)}(0,y) + \frac{1}{s^{k\alpha}} Ł_\alpha\{f(x,y)\} - \frac{1}{s^{k\alpha}} Ł\{R_\alpha(x,y)\}. \tag{6}$$

Taking the inverse of local fractional Laplace transform on Eq.(6), we obtain

$$\begin{aligned} u(x,y) = {} & u(0,y) + \frac{x^\alpha}{\Gamma(1+\alpha)} u^{(\alpha)}(0,y) + \cdots + \frac{x^{(k-1)\alpha}}{\Gamma[1+(k-1)\alpha]} u^{((k-1)\alpha)}(0,y) + Ł_\alpha^{-1}\left(\frac{1}{s^{k\alpha}} Ł_\alpha\{f(x,y)\}\right) \\ & - Ł_\alpha^{-1}\left(\frac{1}{s^{k\alpha}} Ł_\alpha\{R_\alpha u(x,y)\}\right). \end{aligned} \tag{7}$$

We are going to represent the solution in an infinite series given below:

$$u(x,y) = \sum_{n=0}^{\infty} u_n(x,y). \tag{8}$$

Substituting (8) into (7), which give us this result

$$\begin{aligned} \sum_{n=0}^{\infty} u_n(x,y) = {} & u(0,y) + \frac{x^\alpha}{\Gamma(1+\alpha)} u^{(\alpha)}(0,y) + \cdots + \frac{x^{(k-1)\alpha}}{\Gamma[1+(k-1)\alpha]} u^{((k-1)\alpha)}(0,y) + Ł_\alpha^{-1}\left(\frac{1}{s^{k\alpha}} Ł_\alpha\{f(x,y)\}\right) \\ & - Ł_\alpha^{-1}\left(\frac{1}{s^{k\alpha}} Ł_\alpha\left\{R_\alpha \sum_{n=0}^{\infty} u_n(x,y)\right\}\right). \end{aligned} \tag{9}$$

When we compare the left and right hand sides of (9) we obtain the recursive relation:

$$u_0(x,y) = u(0,y) + \frac{x^\alpha}{\Gamma(1+\alpha)} u^{(\alpha)}(0,y) + \cdots + \frac{x^{(k-1)\alpha}}{\Gamma[1+(k-1)\alpha]} u^{((k-1)\alpha)}(0,y) + Ł_\alpha^{-1}\left(\frac{1}{s^{k\alpha}} Ł_\alpha\{f(x,y)\}\right),$$

$$u_{n+1}(x,y) = -Ł_\alpha^{-1}\left(\frac{1}{s^{k\alpha}} Ł_\alpha\{R_\alpha u_n(x,y)\}\right), \ n \ge 0$$

(10)

3. Applications

Example 1. Consider the local fractional Helmholtz equation with local fractional operator

$$\frac{\partial^{2\alpha} u(x,y)}{\partial x^{2\alpha}} + \frac{\partial^{2\alpha} u(x,y)}{\partial y^{2\alpha}} - u(x,y) = 0, \ 0 < \alpha \le 1,$$

(11)

with the initial value conditions as follows:

$$u(0,y)=0,\quad \frac{\partial^{\alpha}u(0,y)}{\partial x^{\alpha}}=\cosh_{\alpha}(y^{\alpha})\ . \tag{12}$$

In view of (10) and (11) the local fractional iteration algorithm can be written as follows:

$$\begin{aligned}&u_0(x,y)=\frac{x^{\alpha}}{\Gamma(1+\alpha)}\cosh_{\alpha}(y^{\alpha}),\\ &u_{n+1}(x,t)=Ł_{\alpha}^{-1}\left(\frac{1}{s^{2\alpha}}Ł_{\alpha}\left\{u_n(x,y)-\frac{\partial^{2\alpha}u_n(x,y)}{\partial y^{2\alpha}}\right\}\right),\ n\geq 0.\end{aligned} \tag{13}$$

Therefore, from (13) we give the components as follows:

$$u_0(x,y)=\frac{x^{\alpha}}{\Gamma(1+\alpha)}\cosh_{\alpha}(y^{\alpha}), \tag{14}$$

$$\begin{aligned}u_1(x,y)&=Ł_{\alpha}^{-1}\left(\frac{1}{s^{2\alpha}}Ł_{\alpha}\left\{u_0(x,y)-\frac{\partial^{2\alpha}u_0(x,y)}{\partial y^{2\alpha}}\right\}\right)\\ &=Ł_{\alpha}^{-1}\left(\frac{1}{s^{2\alpha}}Ł\left\{\frac{x^{\alpha}}{\Gamma(1+\alpha)}\cosh_{\alpha}(y^{\alpha})-\frac{x^{\alpha}}{\Gamma(1+\alpha)}\cosh_{\alpha}(y^{\alpha})\right\}\right)=0,\end{aligned} \tag{15}$$

$$u_2(x,y)=Ł_{\alpha}^{-1}\left(\frac{1}{s^{2\alpha}}Ł_{\alpha}\left\{u_1(x,y)-\frac{\partial^{2\alpha}u_1(x,y)}{\partial y^{2\alpha}}\right\}\right)=0,$$

(16)

and so on, the other components can be found in the similar manner.

Therefore, the solution in series form is given by

$$\begin{aligned}u(x,y)&=u_1(x,y)+u_2(x,y)+u_3(x,y)+\cdots\\ &=\frac{x^{\alpha}}{\Gamma(1+\alpha)}\cosh_{\alpha}(y^{\alpha}).\end{aligned} \tag{17}$$

Example 2. Let us consider the local fractional Helmholtz equation with local fractional operator

$$\frac{\partial^{2\alpha}u(x,y)}{\partial x^{2\alpha}}+\frac{\partial^{2\alpha}u(x,y)}{\partial y^{2\alpha}}+u(x,y)=\frac{x^{\alpha}}{\Gamma(1+\alpha)}\frac{y^{\alpha}}{\Gamma(1+\alpha)},\ 0<\alpha\leq 1, \tag{18}$$

with the initial value conditions as follows:

$$u(0,y)=0,\quad \frac{\partial^{\alpha}u(0,y)}{\partial x^{\alpha}}=\frac{y^{\alpha}}{\Gamma(1+\alpha)}\ . \tag{19}$$

In view of (10) and (18) the local fractional iteration algorithm can be written as follows:

$$\begin{aligned}&u_0(x,y)=\frac{x^{\alpha}}{\Gamma(1+\alpha)}\frac{y^{\alpha}}{\Gamma(1+\alpha)}+Ł_{\alpha}^{-1}\left(\frac{1}{s^{2\alpha}}Ł_{\alpha}\left\{\frac{x^{\alpha}}{\Gamma(1+\alpha)}\frac{y^{\alpha}}{\Gamma(1+\alpha)}\right\}\right),\\ &u_{n+1}(x,t)=-Ł_{\alpha}^{-1}\left(\frac{1}{s^{2\alpha}}Ł_{\alpha}\left\{u_n(x,y)+\frac{\partial^{2\alpha}u_n(x,y)}{\partial y^{2\alpha}}\right\}\right),\ n\geq 0.\end{aligned} \tag{20}$$

Therefore, from (20) we give the components as follows:

$$u_0(x,y)=\frac{x^\alpha}{\Gamma(1+\alpha)}\frac{y^\alpha}{\Gamma(1+\alpha)}+Ł_\alpha^{-1}\left(\frac{1}{s^{4\alpha}}\frac{y^\alpha}{\Gamma(1+\alpha)}\right)=\frac{x^\alpha}{\Gamma(1+\alpha)}\frac{y^\alpha}{\Gamma(1+\alpha)}+\frac{x^{3\alpha}}{\Gamma(1+3\alpha)}\frac{y^\alpha}{\Gamma(1+\alpha)},$$

(21)

$$u_1(x,y)=-Ł_\alpha^{-1}\left(\frac{1}{s^{2\alpha}}Ł_\alpha\left\{u_0(x,y)+\frac{\partial^{2\alpha}u_0(x,y)}{\partial y^{2\alpha}}\right\}\right)$$

$$=-Ł_\alpha^{-1}\left(\frac{1}{s^{2\alpha}}Ł\left\{\frac{x^\alpha}{\Gamma(1+\alpha)}\frac{y^\alpha}{\Gamma(1+\alpha)}-\frac{x^{3\alpha}}{\Gamma(1+3\alpha)}\frac{y^\alpha}{\Gamma(1+\alpha)}\right\}\right)$$

$$=-Ł_\alpha^{-1}\left(\frac{1}{s^{4\alpha}}\frac{y^\alpha}{\Gamma(1+\alpha)}+\frac{1}{s^{6\alpha}}\frac{y^\alpha}{\Gamma(1+\alpha)}\right)$$

$$=-\frac{x^{3\alpha}}{\Gamma(1+3\alpha)}\frac{y^\alpha}{\Gamma(1+\alpha)}-\frac{x^{5\alpha}}{\Gamma(1+5\alpha)}\frac{y^\alpha}{\Gamma(1+\alpha)}, \quad (22)$$

$$u_2(x,y)=-Ł_\alpha^{-1}\left(\frac{1}{s^{2\alpha}}Ł_\alpha\left\{u_1(x,y)+\frac{\partial^{2\alpha}u_1(x,y)}{\partial y^{2\alpha}}\right\}\right)$$

$$=-Ł_\alpha^{-1}\left(\frac{1}{s^{2\alpha}}Ł\left\{-\frac{x^{3\alpha}}{\Gamma(1+3\alpha)}\frac{y^\alpha}{\Gamma(1+\alpha)}-\frac{x^{5\alpha}}{\Gamma(1+5\alpha)}\frac{y^\alpha}{\Gamma(1+\alpha)}\right\}\right)$$

$$=Ł_\alpha^{-1}\left(\frac{1}{s^{6\alpha}}\frac{y^\alpha}{\Gamma(1+\alpha)}+\frac{1}{s^{8\alpha}}\frac{y^\alpha}{\Gamma(1+\alpha)}\right)$$

$$=\frac{x^{5\alpha}}{\Gamma(1+5\alpha)}\frac{y^\alpha}{\Gamma(1+\alpha)}+\frac{x^{7\alpha}}{\Gamma(1+7\alpha)}\frac{y^\alpha}{\Gamma(1+\alpha)}, \quad (23)$$

and so on, the other components can be found in the similar manner.

Therefore, the solution in series form is given by

$$u(x,y)=u_1(x,y)+u_2(x,y)+u_3(x,y)+\cdots$$

$$=\frac{x^\alpha}{\Gamma(1+\alpha)}\frac{y^\alpha}{\Gamma(1+\alpha)}. \quad (24)$$

Example 3. Consider the following local fractional coupled Helmholtz equations with local fractional derivative:

$$\begin{aligned}&\frac{\partial^{2\alpha}u(x,y)}{\partial x^{2\alpha}}+\frac{\partial^{2\alpha}v(x,y)}{\partial y^{2\alpha}}-u(x,y)=0,\\&\frac{\partial^{2\alpha}v(x,t)}{\partial x^{2\alpha}}+\frac{\partial^{2\alpha}u(x,t)}{\partial y^{2\alpha}}-v(x,y)=0,\end{aligned} \quad (25)$$

subject to the initial conditions

$$\begin{aligned}&u(0,y)=0,\ \frac{\partial^{\alpha}u(0,y)}{\partial x^{\alpha}}=E_\alpha(y^\alpha),\\&v(0,y)=0,\ \frac{\partial^{\alpha}u(0,y)}{\partial y^{\alpha}}=-E_\alpha(y^\alpha).\end{aligned} \quad (26)$$

Applying local fractional Laplace transform on Eq. (25) and using the initial conditions, we have

$$\text{Ł}_\alpha\{u(x,y)\} = \frac{1}{s^{2\alpha}} E_\alpha(y^\alpha) + \frac{1}{s^{2\alpha}} \text{Ł}_\alpha \left\{ u(x,y) - \frac{\partial^{2\alpha} v(x,y)}{\partial y^{2\alpha}} \right\},$$
$$\text{Ł}_\alpha\{v(x,y)\} = -\frac{1}{s^{2\alpha}} E_\alpha(y^\alpha) + \frac{1}{s^{2\alpha}} \text{Ł}_\alpha \left\{ v(x,y) - \frac{\partial^{2\alpha} u(x,y)}{\partial y^{2\alpha}} \right\}. \quad (27)$$

Operating with the local fractional Laplace transform inverse on both sides of Eq. (27) we obtain

$$u(x,y) = \frac{x^\alpha}{\Gamma(1+\alpha)} E_\alpha(y^\alpha) + \text{Ł}_\alpha^{-1}\left(\frac{1}{s^{2\alpha}} \text{Ł}_\alpha \left\{ u(x,y) - \frac{\partial^{2\alpha} v(x,y)}{\partial y^{2\alpha}} \right\} \right),$$
$$v(x,y) = -\frac{x^\alpha}{\Gamma(1+\alpha)} E_\alpha(y^\alpha) + \text{Ł}_\alpha^{-1}\left(\frac{1}{s^{2\alpha}} \text{Ł}_\alpha \left\{ v(x,y) - \frac{\partial^{2\alpha} u(x,y)}{\partial y^{2\alpha}} \right\} \right). \quad (28)$$

We now look for solutions $u(x,y)$ and $v(x,y)$ of Eq. (28) having the series form

$$u(x,y) = \sum_{n=0}^{\infty} u_n(x,y),$$
$$v(x,y) = \sum_{n=0}^{\infty} v_n(x,y). \quad (29)$$

In view of Eq. (29), Eq. (28) is equivalent to

$$\sum_{n=0}^{\infty} u_n(x,y) = \frac{x^\alpha}{\Gamma(1+\alpha)} E_\alpha(y^\alpha) + \text{Ł}_\alpha^{-1}\left(\frac{1}{s^{2\alpha}} \text{Ł}_\alpha \left\{ \sum_{n=0}^{\infty} u_n(x,y) - \frac{\partial^{2\alpha}}{\partial y^{2\alpha}} \left[\sum_{n=0}^{\infty} v_n(x,y) \right] \right\} \right),$$
$$\sum_{n=0}^{\infty} v_n(x,y) = -\frac{x^\alpha}{\Gamma(1+\alpha)} E_\alpha(y^\alpha) + \text{Ł}_\alpha^{-1}\left(\frac{1}{s^{2\alpha}} \text{Ł}_\alpha \left\{ \sum_{n=0}^{\infty} v_n(x,y) - \frac{\partial^{2\alpha}}{\partial y^{2\alpha}} \left[\sum_{n=0}^{\infty} u_n(x,y) \right] \right\} \right). \quad (30)$$

We define the recurrence relation:

$$u_0(x,y) = \frac{x^\alpha}{\Gamma(1+\alpha)} E_\alpha(y^\alpha),$$
$$v_0(x,y) = -\frac{x^\alpha}{\Gamma(1+\alpha)} E_\alpha(y^\alpha) \quad (31)$$

$$u_{n+1}(x,y) = \text{Ł}_\alpha^{-1}\left(\frac{1}{s^{2\alpha}} \text{Ł}_\alpha \left\{ u_n(x,y) - \frac{\partial^{2\alpha} v_n(x,y)}{\partial y^{2\alpha}} \right\} \right),$$
$$v_{n+1}(x,y) = \text{Ł}_\alpha^{-1}\left(\frac{1}{s^{2\alpha}} \text{Ł}_\alpha \left\{ v_n(x,y) - \frac{\partial^{2\alpha} u_n(x,y)}{\partial y^{2\alpha}} \right\} \right). \quad (32)$$

Therefore, from (31) and (32) we give the components as follows:

$$u_1(x,y) = \text{Ł}_\alpha^{-1}\left(\frac{1}{s^{2\alpha}} \text{Ł}_\alpha \left\{ u_0(x,y) - \frac{\partial^{2\alpha} v_0(x,y)}{\partial y^{2\alpha}} \right\} \right) = \text{Ł}_\alpha^{-1}\left(\frac{2}{s^{4\alpha}} E_\alpha(y^\alpha) \right) = \frac{2x^{3\alpha}}{\Gamma(1+3\alpha)} E_\alpha(y^\alpha),$$
$$v_1(x,y) = \text{Ł}_\alpha^{-1}\left(\frac{1}{s^{2\alpha}} \text{Ł}_\alpha \left\{ v_0(x,y) - \frac{\partial^{2\alpha} u_0(x,y)}{\partial y^{2\alpha}} \right\} \right) = \text{Ł}_\alpha^{-1}\left(\frac{-2}{s^{4\alpha}} E_\alpha(y^\alpha) \right) = -\frac{2x^{3\alpha}}{\Gamma(1+3\alpha)} E_\alpha(y^\alpha), \quad (33)$$

$$u_2(x,y)=Ł_\alpha^{-1}\left(\frac{1}{s^{2\alpha}}Ł_\alpha\left\{u_1(x,y)-\frac{\partial^{2\alpha}v_1(x,y)}{\partial y^{2\alpha}}\right\}\right)=Ł_\alpha^{-1}\left(\frac{4}{s^{6\alpha}}E_\alpha(y^\alpha)\right)=\frac{4x^{5\alpha}}{\Gamma(1+5\alpha)}E_\alpha(y^\alpha),$$
$$v_2(x,y)=Ł_\alpha^{-1}\left(\frac{1}{s^{2\alpha}}Ł_\alpha\left\{v_1(x,y)-\frac{\partial^{2\alpha}u_1(x,y)}{\partial y^{2\alpha}}\right\}\right)=Ł_\alpha^{-1}\left(\frac{-4}{s^{6\alpha}}E_\alpha(y^\alpha)\right)=-\frac{4x^{5\alpha}}{\Gamma(1+5\alpha)}E_\alpha(y^\alpha), \quad (34)$$

$$u_3(x,y)=Ł_\alpha^{-1}\left(\frac{1}{s^{2\alpha}}Ł_\alpha\left\{u_2(x,y)-\frac{\partial^{2\alpha}v_2(x,y)}{\partial y^{2\alpha}}\right\}\right)=Ł_\alpha^{-1}\left(\frac{8}{s^{8\alpha}}E_\alpha(y^\alpha)\right)=\frac{8x^{7\alpha}}{\Gamma(1+7\alpha)}E_\alpha(y^\alpha),$$
$$v_3(x,y)=Ł_\alpha^{-1}\left(\frac{1}{s^{2\alpha}}Ł_\alpha\left\{v_2(x,y)-\frac{\partial^{2\alpha}u_2(x,y)}{\partial y^{2\alpha}}\right\}\right)=Ł_\alpha^{-1}\left(\frac{-8}{s^{8\alpha}}E_\alpha(y^\alpha)\right)=-\frac{8x^{7\alpha}}{\Gamma(1+7\alpha)}E_\alpha(y^\alpha), \quad (35)$$

and so on, the other components can be found in the similar manner.

Therefore, the series solutions can be written in the form

$$u(x,y)=E_\alpha(y^\alpha)\left(\frac{x^\alpha}{\Gamma(1+\alpha)}+\frac{2x^{3\alpha}}{\Gamma(1+3\alpha)}+\frac{4x^{5\alpha}}{\Gamma(1+5\alpha)}-\cdots\right)=E_\alpha(y^\alpha)\frac{\sinh_\alpha\left(\sqrt{2}x^\alpha\right)}{\sqrt{2}}$$
$$v(x,y)=-E_\alpha(y^\alpha)\left(\frac{x^\alpha}{\Gamma(1+\alpha)}+\frac{2x^{3\alpha}}{\Gamma(1+3\alpha)}+\frac{4x^{5\alpha}}{\Gamma(1+5\alpha)}-\cdots\right)=-E_\alpha(y^\alpha)\frac{\sinh_\alpha\left(\sqrt{2}x^\alpha\right)}{\sqrt{2}}. \quad (36)$$

Conclusions

In this work, the local fractional Laplace decomposition method is demonstrated as an effective method for solving partial differential equations on Cantor sets within local fractional operator. The approximate solutions for local fractional Helmholtz and coupled Helmholtz equations were obtained. The obtained results show that the method is a powerful and an efficient technique in finding the solutions for wide classes of problems.

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